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Longitudinal magnetoresistance of ultrathin films and two-dimensional electron layers

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Abstract. A method of calculating the longitudinal magnetoresistance of ultrathin films or inversion layers in MOS structures with sufficient size quantisation of transverse electron motion is proposed within the framework of the theory of weak localisation. The cases when a single subband and several subbands of size quantisation are filled are discussed. It is shown that in all studied cases the magnetoresistance is negative and depends on the ratio H^2/T, so that it can be distinguished experimentally from the H/T-dependences of electron–electron interaction corrections to conductivity. Extension of the obtained results to the magnetoresistance calculation of quasi-1D channel is also discussed.

1. Introduction

The phenomenon of weak localisation of electrons in disordered conductors determines many features of their low-temperature kinetic properties. For instance, the anomalous magnetic field and temperature dependences of resistivity of dirty metals arise from sensitivity of interference corrections to conductivity to breaking of the time–reversal symmetry. In the theory [1, 2] the origin of these corrections (usually called quantum corrections) to conductivity is explained as a result of interference between two self-intersecting diffusive paths differing only in the direction of traversing the closed loop. The external magnetic field applied to the sample destroys the phase coherence between these two paths and suppresses the quantum correction to conductivity, thus giving rise to a negative magnetoresistance.

In the films which are thick enough to consider the electron motion across the layer as classical, the lines of reasoning in the analysis of the magnetoresistance in two different geometries, i.e. with the magnetic field parallel and perpendicular to the film, are qualitatively similar. Then all estimations can be made by calculating additional phases acquired by an electron in the magnetic field [3–5]. In this paper we address the opposite limit of ultrathin films and study the weak localisation in systems with quantised motion in one direction (the z axis). Both the case of a film with a single subband filled and the intermediate case, when many subbands are filled in a quantum well, are analysed in the first two sections, and the relevant methods of calculating the longitudinal magnetoresistance in these systems are proposed.

2. A film with only the lowest subband filled

In ultrathin films and inversion layers with a quantised transverse motion of carriers the suppression of weak localisation by a parallel magnetic field arises because of some

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features that the latter brings to 2D scattering [6]. The point is that in many experimentally investigated quasi-2D systems the localised scatterers, which cause the momentum relaxation, are located asymmetrically with respect to the middle of a quantum well. In inversion layers in MOS devices the role of such scatterers is played by irregularities of the semiconductor surface, and in films by impurities randomly distributed along the width of a sample.

If the magnetic field is oriented in the plane of a film, it couples the transverse motion and longitudinal motion of electrons, and the transverse part $\varphi_p^{(n)}(z)$ of the full free-carrier wavefunction

$$\Psi_p^{(n)}(z, \boldsymbol{r}) = \exp(i\boldsymbol{p} \cdot \boldsymbol{r})\, \varphi_p^{(n)}(z) \tag{1}$$

acquires a dependence on its longitudinal momentum. The Born amplitude of electron scattering on the asymmetrically located defects is determined by the matrix elements of the three-dimensional scatterer potential $u(x, y, z)$ between wavefunctions (1), and therefore must have this dependence. To describe the character of the effect, it is convenient to assume the scatterer potential to have a separable form $u(x, y, z) = u(x, y)f(z)$, where a dimensionless function $f(z) \sim 1$ is connected with an asymmetry of scattering potential in the z direction. Thus the magnetic field leads to the Born amplitude

$$g_{p,p'} = g^{(0)}_{(p-p')}[1 + \xi(\boldsymbol{p} + \boldsymbol{p}') \cdot (\boldsymbol{H} \times \mathbf{1}_z)] \tag{2}$$

where

$$\xi = \frac{e}{mc} \sum_n \frac{f_{0n} z_{n0}}{f_{00}(E_n - E_0)}$$

containing the term linear in $\boldsymbol{H}$ and the total momentum $\boldsymbol{p} + \boldsymbol{p}'$, where $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the electron momenta before and after the collision respectively and $\mathbf{1}_z$ is a unit vector in the z direction. Phenomenologically, this special term in (2) results from the simultaneous breaking of two symmetries in the system, i.e. from the breaking of the time-reversal symmetry caused by the magnetic field and the inversion symmetry breaking due to asymmetrical location of a scatterer. In further considerations the parameter ξ used to define the degree of asymmetry of a scatterer can be treated as phenomenological, and its value for MOS structures can be extracted from the magnitude of the photomagnetic effect [6].

On the other hand, the special form of the amplitude (2) is the consequence of the initial 3d character of electron motion but, all in all, these carriers could be regarded as almost 2D particles. Therefore in the qualitative analysis of efficiency of weak localisation we shall follow the usual method used in calculations of quantum corrections to conductivity [1] in its two-dimensional version. This consists in estimating the portion of closed paths

$$\frac{\delta\sigma_2}{\sigma} = \int_\tau^\infty \mathrm{d}t \frac{\lambda_F v_F}{Dt} \exp\left(-\frac{t}{\tau_\varphi}\right). \tag{3}$$

That can contribute effectively to the interference enhancement of the probability of return among all possible diffusive paths. The exponential factor in (3) accounts for inelastic processes which destroy the phase coherence between pairs of time-reversed trajectories.

In the estimation (3) the condition of equality of the realisation probabilities $w_{1,2}$ of two self-intersecting diffusive paths traversed in opposite directions is used. In the case in point this condition is not satisfied. Two probabilities mentioned above are determined

by the product of scattering amplitudes in each inflection of a diffusive path. Because of the special form of the Born amplitude (2), the parallel magnetic field gives rise to a number of additional multipliers $1 \pm \xi(\boldsymbol{p}+\boldsymbol{p}') \cdot (\boldsymbol{H} \times \mathbf{1}_z)$ in $w_{1,2}$ with random values of the sum $\boldsymbol{p}+\boldsymbol{p}'$. Thus the averaged total probability of passing a closed loop in opposite directions undergoes exponential suppression:

$$w(\boldsymbol{H}) \sim w_1(\boldsymbol{H}) w_2(\boldsymbol{H}) \sim \left\langle \prod_{t/\tau} \{1 - [\xi(\boldsymbol{p}+\boldsymbol{p}') \cdot (\boldsymbol{H} \times \mathbf{1}_z)]^2\} \right\rangle w(0)$$
$$\sim \exp(-2\xi^2 p_F^2 H^2 t/\tau)\, w(0) \tag{4}$$

when the number t/τ of elastic collisions, which the particle suffers until it returns to the point of departure, is large enough. In equation (4), τ denotes the mean free path time in the system in the absence of a magnetic field.

Now we see that field-induced relaxation arises, and the relaxation rate

$$\tau_H^{-1} = 2\xi^2 p_F^2 H^2 \tau^{-1} \sim (Hd^2/\Phi_0)^2 (p_F d/\hbar)^2 \tau^{-1} \qquad p_F d < \hbar \tag{5}$$

must be added to the inelastic phase relaxation rate τ_φ^{-1} in (3). In (5), d is the efficient thickness of electron layer. A simple calculation of the integral (3) shows that the magnetoresistance

$$\Delta\sigma_2 = \delta\sigma_2(H) - \delta\sigma_2(0) = (e^2/h) \ln(1 + \tau_\varphi/\tau_H) \tag{6}$$

in the system under investigation is negative and obeys the quadratic and logarithmic laws in weak and strong magnetic fields respectively. The dependences of the relaxation rate (5) on the magnetic field and the parameters of the 2D electron gas have already been clarified in the text. The numerical coefficient in (5) is a result of more rigorous calculation expounded in the appendix.

3. A film with a few subbands filled

In thin films with a few subbands filled, the terms linear in $\boldsymbol{H}$ and $\boldsymbol{p}+\boldsymbol{p}'$ also arise in all amplitudes of intersubband scattering. Thus the phase relaxation rate τ_H^{-1} must have an additional factor $p_F d/\hbar$ in comparison with that calculated above. To obtain this result, one can follow the method described in the appendix. Nevertheless, in the present paper we prefer another line of reasoning based on the quasiclassical treatment of the problem, because it permits us to separate the regions of film parameters, where the size quantisation is sufficient and where it is not.

Consider electron motion in a rather pure metal or a semiconductor layer of width d somewhat less than the mean free path l. The 3D quasiclassical trajectory of a particle in such a film has the form of a series of ballistic tracks from one surface to the other with specular boundary reflection and an inclination angle α unchanged between two impurity collisions. Since all closed paths involving only wall collisions enclose zero flux, only rare events of impurity scattering are responsible for the additional phase that an electron acquires in a magnetic field. As a result, the suppression of the phase correlations

$$K(t) = \langle \exp\{i\varphi(t)\} \rangle = \exp\left[-\frac{t}{2\tau} \int_{\alpha_0} \mathrm{d}\alpha \left(\frac{eHd^2}{c\alpha} \right)^2 \right] \tag{7}$$

occurs when $t > \tau_H$. The dephasing time τ_H is determined by the mean square value of the characteristic shaded areas in figure 1 and can be expressed in terms of a minimal

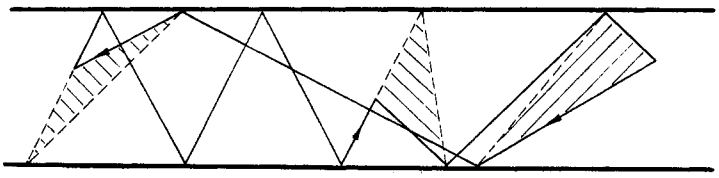

Figure 1. The characteristic form of an electron trajectory in a thin film of pure metal. The shaded areas are responsible for additional phase acquired by an electron in the magnetic field.

inclination angle α_0 of the ballistic electron path across the layer. As this angle $\alpha_0 \sim \hbar/p_F d$ is determined by the ratio of the transverse momentum in the lowest occupied subband to the Fermi momentum p_f, the phase relaxation rate can be written as

$$\tau_H^{-1} \sim (p_F d/\hbar)\,(Hd^2/\Phi_0)^2 \tau^{-1} \qquad p_F d > \hbar \tag{8}$$

and depends on the elastic scattering time τ. If the angle α_0 is limited by impurity collisions, it equals the ratio d/l and the corresponding magnetic field phase relaxation rate [4]

$$\tau_H^{-1} = (Hd^2/\Phi_0)^2 v_F/d \tag{9}$$

depends only on the width d of a sample and the Fermi velocity v_F, but not on the momentum relaxation rate.

The crossover between two asymptotical regimes (8) and (9) takes place when the angles α_0 defined by the two conditions mentioned above are equal. It is convenient to follow this crossover in figure 2, where all main regions of film parameters and the respective forms of the phase relaxation rate in the magnetic field are shown. Thus the purely classical description (9) is suitable for films of width

$$d > \sqrt{l\lambda_F} \tag{10}$$

which is parametrically larger than the Fermi wavelength of electrons. To evaluate the longitudinal magnetoresistance of somewhat thinner films, one ought to substitute τ_H from (8) into the general expression (5). The applicability of the latter estimation extends up to films with $d \sim \lambda_F$, when the extracted multiplyer $p_F d/\hbar$ in (8) must be replaced by unity. A further decrease in the sample thickness leads to an additional small prefactor $(p_F d/\hbar)^2$ in the dephasing rate $(Hd^2\Phi_0)^2(p_F d/\hbar)^2\tau^{-1}$, and one returns to the case described by (6).

The relaxation time τ_H in the opposite case of films with diffusive electron motion across a sample corresponds to that shown on the right-hand side of figure 2. The parameter $p_F l/\hbar$, which permits us to separate the regimes (8) and (9), is suggested to be large, as is always done when considering weak localisation.

$$\begin{array}{ccccccc} \dfrac{(p_F d/\hbar)^2}{\tau} & & \dfrac{p_F d/\hbar}{\tau} & & \dfrac{v_F}{d} & \dfrac{v_F l}{d^2} & \sim \left[\dfrac{\Phi_0}{Hd^2}\right]^2 \tau_H^{-1} \\ \hline 1 & & \sqrt{p_F l} & & p_F l & & p_F d \end{array}$$

Figure 2. Different regions of film parameters and the relevant forms of the phase relaxation rate.

4. Conclusions

In all asymptotical regimes studied in this paper the field and temperature dependences of the magnetoresistance are described by the function of the single combination $\tau_\varphi(T)/\tau_H(H)$. In thin metallic films and inversion layers in MOS structures at low temperatures, $\tau_\varphi^{-1} \sim T^{1+\delta}$, $\delta \ll 1$ [7]; thus $\Delta\sigma_2(H, T) \simeq \Delta\sigma_2(\beta H^2/T)$. The dependences of this kind have already been observed in p-MOS devices [8]. It is necessary to note that these dependences are characteristic of the proposed mechanism of the magnetoresistance, because expressions for electron–electron interaction corrections and spin-orbital corrections contain magnetic field and temperature in the form $\mu H/T$ [2, 9]. As to the sign of the effect, the calculated magnetoresistance is negative in contrast to field-induced resistance changes in electron–electron interaction corrections. Thus the interaction mechanism of the magnetoresistance predominates at lower temperatures, while the weak-localisation corrections correspond to $T \geqslant \mu^2/\beta$ (with β taken from (6), (8) or (9)).

Finally, it is useful to note that the proposed method of evaluation of the phase relaxation times τ_H can be applied to studies of *the magnetoresistance of quantum wires* in the usual geometry. As τ_H^{-1} is represented by (8),

$$\Delta\sigma_1(H) = 2(e^2/h)(D\tau_\varphi)^{1/2}[(1 - \tau_\varphi/\tau_H)^{-1/2} - 1] \tag{11}$$

and the crossover (10) between quantum and classical description of electron transverse motion remains valid.

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Appendix

To calculate the quantum correction to conductivity more accurately we shall use its expression

$$\delta\sigma = \frac{e^2}{hD}\int \mathrm{d}\boldsymbol{Q}\, C(\boldsymbol{Q}, \Omega = 0) \tag{A1}$$

in terms of two-particle Green functions (cooperons) [1] $C(\boldsymbol{Q}, \Omega, \boldsymbol{p}, \boldsymbol{p}')$, which denote the sum of the diagrams shown in figure A1. According to the particular notation of the diagrammatic technique, the crosses in these graphs symbolise electron interactions with impurities and correspond to respective scattering amplitudes in the Born approximation. In the present calculation the renormalisation of Born amplitudes by the magnetic field

$$g_{p,p'} = g^{(0)}_{(p-p')}\{1 + \xi(\boldsymbol{p} + \boldsymbol{p}')\cdot\mathbf{1}_H + \eta[(\boldsymbol{p} + \boldsymbol{p}')\cdot\mathbf{1}_H]^2 + \ldots\} \qquad \mathbf{1}_H = [\boldsymbol{H}\times\mathbf{1}_z] \tag{A2}$$

takes into account not only linear terms (as in (2)) but also corrections quadratic in $\boldsymbol{H}$. This renormalisation is sumultaneously incorporated into the momentum dependence of the momentum relaxation rate

$$\tau^{-1}(\boldsymbol{p}) = \tau^{-1}\{1 + 2\xi(\boldsymbol{p}\cdot\mathbf{1}_H) + (\xi^2 + 2\eta)[(\boldsymbol{p}\cdot\mathbf{1}_H)^2 + H^2 p_F^2/2]\} \tag{A3}$$

and therefore is included in the single-particle Green functions $G^{R(A)} = [\varepsilon - E(p) \pm i\tau^{-1}(\boldsymbol{p})/2]^{-1}$ (which are depicted by the full lines in figure A1).

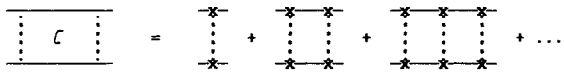

Figure A1. The perturbation theory series used to calculate the cooperon.

The summation of the perturbation theory series in figure A1 can be reduced [1] to solution of a diffusion-like equation for the cooperon so that, in the absence of the longitudinal field, $C = (-i\Omega + DQ^2 + \tau_\varphi^{-1})^{-1}$. The magnetic field introduces a more complicated dependence of C on the initial momentum $\boldsymbol{p}$ and final momentum $\boldsymbol{p}'$. However, until the field is weak, we may expand C into power series in $\boldsymbol{p} \times \mathbf{1}_H$ and $\boldsymbol{p} \times \mathbf{1}_H$, and the cooperon acquires the form

$$C = [1 + (2\eta - \xi^2)(l|\boldsymbol{p} \times \mathbf{1}_H|^2 + |\boldsymbol{p}' \times \mathbf{1}_H|^2)]C_1(Q) \tag{A4}$$

where C_1 satisfies a diffusion-like equation with an additional relaxation term

$$\tau_H^{-1} = 2\xi^2 p_F^2 H^2/\tau. \tag{A5}$$

As all main corrections of the first and the second order in H are accounted for in (A4), and the phase relaxation rate (A5) arises solely from the linear term in (A2), the present calculation is adequate for the qualitative discussion in the text.

In addition, it is useful to note that the special character of scattering in our calculation does not change the form of the particle–hole Green function (diffusion), which accords with the result of the analysis [6] of the kinetic equation with the collision integral containing (A2).

References

[1] Altshuler B L, Aronov A G, Khmel'nitskii D E and Larkin A I 1982 *Quantum Theory of Solids* ed I M Lifshits (Moscow: Mir) p 130
[2] Lee P A and Ramakrishnan T V 1985 *Rev. Mod. Phys.* **57** 287
[3] Altshuler B L and Aronov A G 1981 *JETP Lett.* **33** 499
[4] Dugaev V C and Khmel'nitskii D E 1984 *Sov. Phys.–JETP* **59** 1038
[5] Beenakker C W J and van Houten H 1988 *Phys. Rev.* B **38** 3232
[6] Fal'ko V I 1989 *Fiz. Tverd. Tela* **31** 39
[7] Altshuler B L and Aronov A G 1985 *Electron–Electron Interactions in Disordered Systems* ed V M Agranovich and A A Maradudin (Amsterdam: North-Holland) p 1
[8] Dolgopolov V T, Dorozhkin S I and Shashkin A A 1984 *Solid State Commun.* **50** 273
[9] Maekava S and Fukuyama H 1981 *J. Phys. Soc. Japan* **50** 4717